Newton School of Technology

NSAT

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NSAT Exam Guidelines

Exam Protocol

- Arrive at least 15 minutes early (by 2:45 PM) for exam setup and proctoring checks.
- All three 3 sections are mandatory and include a sectional cutoff that determines passing eligibility
- The exam will be proctored. Unfair means will lead to permanent disqualification.
- For MCQ-type Questions, Positive marking will be (+4) for correct answers, while negative marking (-1) is applicable for incorrect ones.
- There is no negative marking in coding section.

Joining Details & Device Restrictions

- Use a laptop/PC with screen-sharing and microphone access
- Dual-camera setup required:
  - Primary camera (webcam) facing your face
  - Secondary camera (mobile/external) showing your workspace
- Mobile phones are only allowed as a secondary camera (not for taking the test)
- Calculators are prohibited; use pen and paper for rough work
- Access the test portal at https://my.newtonschool.co/nsat/timeline
- Accept the calendar invite sent to you on test day as a reminder
- Maintain a stable internet connection to prevent interruptions during NSAT
- Keep both cameras ON throughout the test
- Position cameras properly:
  - Primary camera: face clearly visible
  - Secondary camera: desk, hands, and screen visible
- Ensure the secondary camera is placed steadily (not handheld)

Preparation

- Visit NSAT homepage to check compatibility before the exam.
- Use Google Chrome for optimal performance.
- Bring a pen and paper for rough work during the exam.
- Attempt a mock test to understand the exam pattern, for practice, and to confirm your PC compatibility.

Environment Considerations

- Sit in a quiet environment with no background noise to minimize distractions and avoid disqualification.
- Ensure the lighting in the room is appropriate for clear visibility and comfortable reading and writing.

SEQUENCE AND SERIES

Sequence:

A sequence is an ordered set of numbers that follows a well-defined pattern or rule. Each number in a sequence is called a term. Understanding sequences is essential for solving complex problems in algebra, calculus, and even physics. This can be classified as follows:

1. **Arithmetic Progression (AP):**
   An arithmetic progression is a sequence where each term is obtained by adding a fixed number (common difference) to the previous term.
2. **Geometric Progression (GP):**
   A geometric progression is a sequence where each term is obtained by multiplying the previous term by a fixed number (common ratio).
3. **Harmonic Progression (HP):**
   A harmonic progression is a sequence where the reciprocals of the terms form an arithmetic progression.

Formulae related to progressions			
	Arithmetic Progression	Geometric Progression	Harmonic Progression
Key Terms	1st term: a Common difference: d	1st term: a Common ratio: r	1st term: a Common difference: $\frac{1}{a_{n-1}}-\frac{1}{a_n}$
Consecutive terms	$a, a+d, a+2d, \ldots, a+nd, \ldots$	$a, ar, ar^2, \ldots, ar^n, \ldots$	$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots, \frac{1}{a+nd}, \ldots$
n^{th} Term	$t_n = a+(n-1)d$	$t_n = ar^{n-1}$	$t_n = \frac{1}{a+(n-1)d}$
Number of terms n	$n = \frac{l-a}{d}+1$	-------	-------
Sum of n terms	$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ Or	$S_n = \frac{a(1-r^n)}{1-r}, r \neq 1$ $S_n = na, r=1$ $S = \frac{a}{1-r}$, for infinite terms $(\lvert r\rvert<1)$	$S_n = \frac{1}{d}\ln\left[\frac{2a+(2n-1)d}{2a-d}\right]$

	$S_n = \frac{n}{2}[a+l]$ $l =$ Last term.		
Mean	$\frac{a_1+a_2+a_3+.......+a_n}{n}$	$\sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot .... \cdot a_n}$	$\frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+.......+\frac{1}{a_n}}$
Consecutive terms a,b,c	$b=\frac{a+c}{2}$	$b^2=ac$	$b=\frac{2ac}{a+c}$
Selection of consecutive terms	3 terms: $a-d,a,a+d$ 4 terms: $a-3d,a-d,a+d,a+3d$	3 terms: $\frac{a}{r},a,ar$ 4 terms: $\frac{a}{r^2},\frac{a}{r},ar,ar^2$.	-----

AM-GM inequality:

The AM-GM inequality states that for any set of positive numbers $a_1,a_2,a_3,....,a_n$:

$$\frac{a_1+a_2+a_3+.......+a_n}{n} \geq \sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot .... \cdot a_n}.$$

Properties of Sequence:

1. The sum of the terms equidistant from the beginning and the end of an A.P. is equal to the sum of the first and last terms.
2. If each term in an A.P. is multiplied, divided, added or subtracted from by some non-zero constant number, then the obtained series will also be an A.P.
3. If we select terms in a regular interval from an A.P., these selected terms will also be in AP.
4. If for an A.P. p^{th} term is q, q^{th} term is p, then m^{th} term is $p+q-m$.
5. The product of the terms equidistant from the beginning and the end of a G.P. is equal to the product of the first and last term.
6. If each term in a G.P. is multiplied or divided by some non-zero constant number, then the obtained series will also be in G.P. Addition or subtraction does not let the G.P. remain a G.P.

7. If the number of terms in AP/GP/HP is odd, then its mid-term is the A.M/G.M/H.M between the first and last number.
8. If the number of terms in AP/GP/HP is even, then the A.M/G.M/H.M of its two middle numbers is the A.M/G.M/H.M between the first and last number.
9. The AM-GM inequality states that for any set of positive numbers $a_1, a_2, a_3, ..., a_n$: A.M≥G.M≥H.M . This is true when the numbers are equal.

Arithmetico-Geometric Progression:

An Arithmetico-Geometric Progression (AGP) is a sequence in which each term is the product of the corresponding terms of an arithmetic progression (A.P.) and a geometric progression (G.P.). The terms are of the form $a, (a+d)r, (a+2d)r^2, ..., (a+(n-1)d)r^{n-1}, ...$

The sum of n terms in an AGP is $S_n = \frac{a}{1-r} + \frac{r.d\left(1-r^{n-1}\right)}{(1-r)^2}$.

For infinite terms, the sum is $S_\infty = \frac{a}{1-r} + \frac{r.d}{(1-r)^2}$.

Fibonacci Sequence

The **Fibonacci Sequence** is defined as the sequence of numbers in which each number in the sequence is equal to the sum of the two preceding numbers. The Fibonacci Sequence can be expressed as: 0,1,1,2,3,5,8,13,21........It has applications in the analysis of financial markets, number theory, algebra, and computer algorithms like searching and sorting.

Series:

A **series** is the sum of the terms of a sequence. That is $a_1 + a_2 + \cdots + a_n + \cdots$

By the sigma notation, $\sum_{i=1}^{n} a_n$ or $\sum_{i=1}^{\infty} a_n$.

Understanding series is crucial for solving advanced problems in algebra, calculus, and mathematical analysis. Series can be classified into different types based on their structure and properties.

1. **Arithmetic Series (A.S.)**: The sum of terms of an arithmetic progression.
2. **Geometric Series (G.S.)**: The sum of terms of a geometric progression.
3. **Harmonic Series (H.S.)**: The sum of terms of a harmonic progression.

For a decreasing H.P., the sum grows very slowly, often diverging logarithmically.

4. **Telescoping Series**: A series where consecutive terms cancel each other out, simplifying the sum significantly.
   - Example: $S_n = \sum_{k=1}^{n}(a_k - a_{k+1})$. After cancellation, we get: $S_n = a_1 - a_{n+1}$.
5. **Exponential Series**:
   - The exponential series represents the power series expansion of e^x, defined as: $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$
   - This series converges for all real and complex values of x and plays a crucial role in calculus and differential equations.
6. **Logarithmic Series**:
   - The logarithmic series is the power series expansion of $\ln(1+x)$, given by: $\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$, for $|x|<1$.
   - It is useful in approximations and numerical calculations of logarithms.

General formulas:

1. Sum of the first n natural numbers: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
2. Sum of the first n odd natural numbers: $\sum_{i=1}^{n}(2i-1) = n^2$.
3. Sum of the first n even natural numbers: $\sum_{i=1}^{n} 2i = n(n+1)$.
4. Sum of squares of the first n natural numbers: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.
5. Sum of cubes of the first n natural numbers: $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$.

Example 1: For $x,y,z>0$, if $x+y+z=7$, then the maximum value of $x^3y^2z^2$ is:

Solution:

$x+y+z$ can be rewritten as $\frac{x}{3}+\frac{x}{3}+\frac{x}{3}+\frac{y}{2}+\frac{y}{2}+\frac{z}{2}+\frac{z}{2}$

By the AM-GM inequality, we have,

$$\frac{\frac{x}{3}+\frac{x}{3}+\frac{x}{3}+\frac{y}{2}+\frac{y}{2}+\frac{z}{2}+\frac{z}{2}}{7}\geq\sqrt[7]{\frac{x}{3}\cdot\frac{x}{3}\cdot\frac{x}{3}\cdot\frac{y}{2}\cdot\frac{y}{2}\cdot\frac{z}{2}\cdot\frac{z}{2}}$$

$$\left(\frac{7}{7}\right)^7\geq\frac{x^3y^2z^2}{432}$$

$$432\geq x^3y^2z^2$$

The maximum value of $x^3y^2z^2$ is 432.

Example 2: If the $3^{rd}, x^{th}$, and 11^{th} terms of an arithmetic progression form a geometric progression, and the 29^{th} term of the arithmetic progression is the next term in the geometric progression, then find the value of x.

Solution:

Let r be the common ratio of the G.P. Then we have,

$$a_A+2d=a$$
$$a_A+(x-1)d=ar$$
$$a_A+10d=ar^2$$
$$a_A+28d=ar^3$$

And,

$$\frac{ar^3-ar^2}{ar^2-ar}=\frac{ar^3-ar}{ar^2-a}$$

So, we have,

$$\frac{a_A+28d-(a_A+10d)}{a_A+10d-(a_A+(x-1)d)}=\frac{a_A+28d-(a_A+(x-1)d)}{a_A+10d-(a_A+2d)}$$
$$\frac{18d}{(11-x)d}=\frac{(29-x)d}{8d}$$
$$(29-x)(11-x)=144$$
$$x^2-40x+175=0$$

Solving further,

$$(x-5)(x-35)=0$$
$$x=5,35$$

Since 5 is between 3 and 11, the value of x is 5.

Example 3: Find the sum of the series $S=\sum_{n=1}^{\infty}\frac{n}{11^n}$.

Solution:

We have,

$$S=\frac{1}{11}+\frac{2}{11^2}+\frac{3}{11^3}+\frac{4}{11^4}+....$$

Then,

$$\frac{S}{11}=\frac{1}{11^2}+\frac{2}{11^3}+\frac{3}{11^4}+......$$

And,

$$S-\frac{S}{11}=\frac{1}{11}+\frac{1}{11^2}+\frac{1}{11^3}+\frac{1}{11^4}+.....$$
$$\frac{10S}{11}=\frac{\frac{1}{11}}{1-\frac{1}{11}}$$
$$\frac{10S}{11}=\frac{1}{10}$$
$$S=\frac{11}{100}$$

Example 4: Find the sum of the series $\sum_{k=1}^{100} \ln\left(\frac{\sqrt{k+1}+1}{\sqrt{k}+1}\right)$.

Solution:
The given series can be rewritten as

$$\sum_{k=1}^{99} \ln\left(\frac{\sqrt{k+1}+1}{\sqrt{k}+1}\right) = \ln\left(\frac{\sqrt{1+1}+1}{\sqrt{1}+1}\right) + \ln\left(\frac{\sqrt{2+1}+1}{\sqrt{2}+1}\right) + \ln\left(\frac{\sqrt{3+1}+1}{\sqrt{3}+1}\right) + \ldots + \ln\left(\frac{\sqrt{99+1}+1}{\sqrt{99}+1}\right)$$

$$= \ln\left(\frac{\sqrt{2}+1}{2} \times \frac{\sqrt{3}+1}{\sqrt{2}+1} \times \frac{\sqrt{4}+1}{\sqrt{3}+1} \times \ldots \times \frac{10+1}{\sqrt{99}+1}\right)$$

$$= \ln\left(\frac{11}{2}\right)$$

$$= \ln(5.5)$$

Example 5: The general term of a Fibonacci sequence is of the form $F(n) = F(n-1) + F(n-2)$. Let S be the set of the first 100 elements of the Fibonacci sequence with the first two elements as 0,1 respectively. The number of elements of the set S that is either divisible by 5 or 7.

Solution:
Since the first 2 elements are 0,1,

$$F(1) = 0$$
$$F(2) = 1$$

The first few elements of the Fibonacci sequence are
0,1,1,2,3,5,8,13,21,34,55,89,144,233,...

Dividing each element by 5 gives the remainders
0,1,1,2,3,0,3,3,1,4,0,4,4,3,..

It can be observed that the remainder is 0 at the positions 1,6,11,...
This pattern continues. So, the number of elements in S that are divisible by 5 will be

$$1 + (n-1)5 \le 100$$
$$n \le 20.8$$

Consider the sequence again.
0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987...

Dividing each element by 7 gives the remainders
$0,1,1,2,3,5,1,6,0,6,6,5,4,2,6,1,0,...$

It can be observed that the remainder is 0 at the positions $1,9,17,25,...$

So, the number of elements in S that are divisible by 7 will be

$$1+(n-1)8 \le 100$$
$$n \le 13.4$$

The first common position is 1.
LCM of the common differences 5,8 is 40.
So, the common difference for the common position terms is 40.
The number of common terms is

$$1+(n-1)40 \le 100$$
$$n \le 3.5$$

The number of elements of the set S that is either divisible by 5 or 7 is $20+13-3=30$.

EXERCISE

1. Let a_n be the n^{th} term of a G.P. And, $\sum_{n=1}^{\infty} a_{3n-1}=25$, $\sum_{n=1}^{\infty} a_{3n}=5$, $\sum_{n=1}^{\infty} a_{3n+1}=1$. Then find $\sum_{n=1}^{\infty} a_n$
2. A 4096-player gaming tournament eliminates half the players in each round. However, a second-chance rule brings back 50% of the eliminated players (rounded to the nearest integer) in every round. When the tournament reaches the player count of 729, the elimination becomes two-thirds, and the second-chance rule becomes 0%. If the tournament ends when only 1 player remains, find the total number of rounds played.
3. Find the sum of the series $\sum_{n=1}^{\infty} \cos\left(\frac{1-2k-4k^2}{1-4k^2}\right)\sin\left(\frac{1}{1-4k^2}\right)$.

SOLUTIONS

1. Since a_n forms a G.P, a_{3n-1} also forms a G.P.

And, since the infinite sum of a G.P is finite only when $|r|<1$,

we have $|r|<1$

From the given data, we have,

$$\sum_{n=1}^{\infty} a_{3n-1} = 25$$
$$a_2 + a_5 + a_8 + \ldots.. = 25$$
$$ar\left(1 + r^3 + r^6 + \ldots\right) = 25$$
$$\frac{ar}{1 - r^3} = 25$$

And,

$$\sum_{n=1}^{\infty} a_{3n} = 5$$
$$a_3 + a_6 + a_9 + \ldots.. = 5$$
$$ar^2\left(1 + r^3 + r^6 + \ldots\right) = 5$$
$$\frac{ar^2}{1 - r^3} = 5$$

Dividing the first and the second equation, we get, $r = \frac{1}{5}$.

And,

$$\frac{a\left(\frac{1}{5}\right)}{1 - \frac{1}{125}} = 25$$
$$a = 124$$

So, we have, $\sum_{n=1}^{\infty} a_n = 155$.

2. If the tournament begins with a, the next round will have

$$\frac{a}{2} + 50\% \text{ of } \frac{a}{2}(\text{Eliminated}) = \frac{a}{2} + \frac{a}{4}$$
$$= \frac{3a}{4}$$

This is a G.P with $a = 4096, r = \frac{3}{4}$.

729 player count is reached when

$$4096\left(\frac{3}{4}\right)^{n-1}=729$$
$$\left(\frac{3}{4}\right)^{n-1}=\left(\frac{3}{4}\right)^{6}$$
$$n=7$$

So, after 6 rounds, the number of players is 729 for the next round.
Then, in each round, one-third of the players move to the next round.

One player remains when $\frac{729}{3^r}=1$.

So, $r=6$.
The number of rounds is $6+6=12$.

3. The series can be rewritten as

$$\sum_{n=1}^{\infty}\cos\left(\frac{1-2k-4k^2}{2-8k^2}\pi\right)\sin\left(\frac{1}{2-8k^2}\pi\right)$$
$$=\sum_{n=1}^{\infty}\frac{\sin\left(\frac{1}{2-8k^2}\pi+\frac{1-2k-4k^2}{2-8k^2}\pi\right)+\sin\left(\frac{1}{2-8k^2}\pi-\frac{1-2k-4k^2}{2-8k^2}\pi\right)}{2}$$
$$=\frac{1}{2}\sum_{n=1}^{\infty}\left[\sin\left(\frac{2(1-2k)(k+1)}{2(1-2k)(1+2k)}\pi\right)+\sin\left(\frac{2k(1+2k)}{2(1-2k)(1+2k)}\pi\right)\right]$$
$$=\frac{1}{2}\sum_{n=1}^{\infty}\left[\sin\left(\frac{k+1}{2k+1}\pi\right)+\sin\left(\frac{-k}{2k-1}\pi\right)\right]$$
$$=\frac{1}{2}\sum_{n=1}^{\infty}\left[\sin\left(\frac{k+1}{2k+1}\pi\right)-\sin\left(\frac{k}{2k-1}\pi\right)\right]$$

Simplifying further, we get,

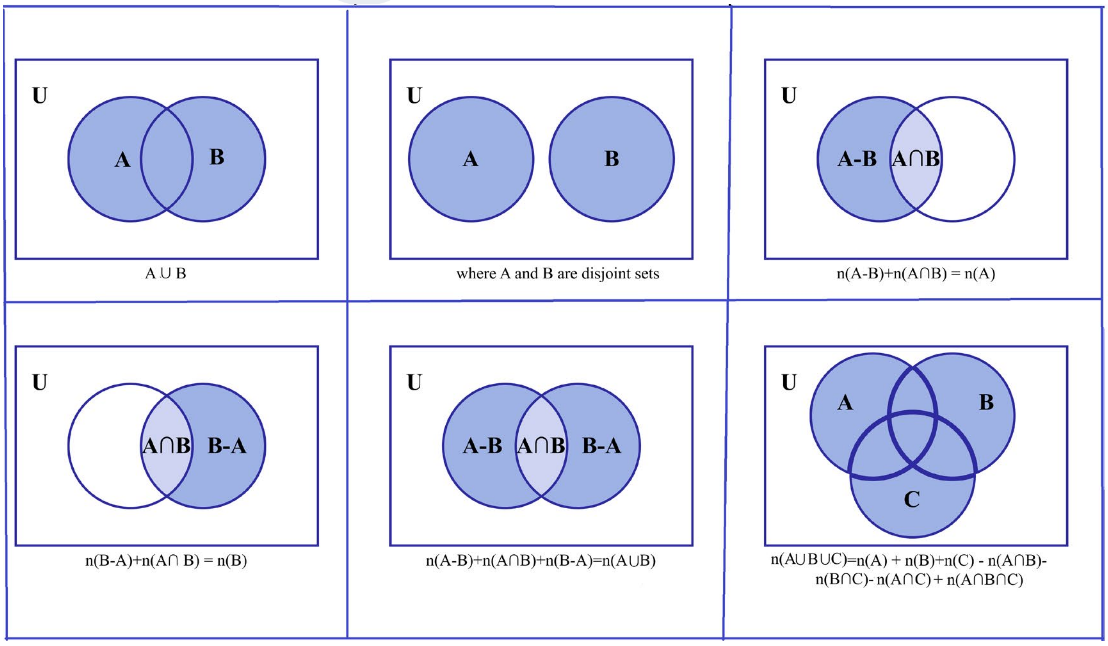
$$\sum_{n=1}^{\infty}\cos\left(\frac{1-2k-4k^2}{2-8k^2}\pi\right)\sin\left(\frac{1}{2-8k^2}\pi\right)$$
$$=\lim_{n\to\infty}\frac{1}{2}\left[\sin\left(\frac{2}{3}\pi\right)-\sin\left(\frac{1}{1}\pi\right)+\sin\left(\frac{3}{5}\pi\right)-\sin\left(\frac{2}{3}\pi\right)+\ldots\ldots..+\sin\left(\frac{n+1}{2n+1}\pi\right)-\sin\left(\frac{n}{2n-1}\pi\right)\right]$$
$$=\lim_{n\to\infty}\frac{1}{2}\left[-\sin(\pi)+\sin\left(\frac{n+1}{2n+1}\pi\right)\right]$$
$$=\frac{1}{2}\left[-0+\lim_{n\to\infty}\sin\left(\frac{1+\frac{1}{n}}{2+\frac{1}{n}}\pi\right)\right]$$
$$=\frac{1}{2}\left[\sin\left(\frac{\pi}{2}\right)\right]$$

So, $\sum_{n=1}^{\infty}\cos\left(\frac{1-2k-4k^2}{2-8k^2}\pi\right)\sin\left(\frac{1}{2-8k^2}\pi\right)=\frac{1}{2}$.

SETS

Set: A set in mathematics is a clearly defined group of distinct items that are regarded as independent objects. Such objects could be letters, numbers, humans, or other intangible entities. Curly braces, like $\{a,b,c\}$, indicate sets, and each item in a set is termed as an element or member. Set theory finds application in probability, Boolean algebra and statistics.

Operations on sets:



Properties of set:

Different Properties	Formulas
Associative Property	$A\cup(B\cup C)=(A\cup B)\cup C$ $A\cap(B\cap C)=(A\cap B)\cap C$
Distributive Property	$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$
De Morgan's Law	Law of union: $(A\cup B)'=A'\cap B'$ Law of intersection: $(A\cap B)'=A'\cup B'$

Relation: A relation is a linear operation that, in accordance with a specific relationship rule, creates a relationship between the components of two sets $(R\subseteq A\times B)$.

Types of Relations:

Different Relations	Formulas
Universal Relations	$R = A \times A$
Identity Relations	$I = \{(a, a) : \forall a \in A\}$
Reflexive relations	$(a, a) \in R \ \forall a \in A$
Symmetric Relations	$(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$
Transitive Relations	$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$
Equivalence relation	A relation i.e. reflexive, symmetric and transitive

Example 1: In a group of 200 scholars, 116 people speak Gujarati and 150 people speak Marathi. Each scholar in the group speaks at least one of the two languages. The number of scholars speaking only Gujarati is 10α and the number of scholars speaking only Marathi is 7β, then what is the linear eccentricity of the hyperbola $\alpha^2 y^2 = \beta^2 x^2 - \alpha^2 \beta^2$?

Solution:

The number of scholars speaking both Gujarati and Marathi is:

$$n(G \cap M) = n(G) + n(M) - n(G \cup M) = 116 + 150 - 200 = 66$$

The number of scholars speaking only Gujarati is,

$$Only\ Gujarati : n(G) - n(G \cap M) = 116 - 66 = 50$$

The number of scholars speaking only Marathi is,

$$Only\ Marathi : n(M) - n(G \cap M) = 150 - 66 = 84$$

The value of α and β will be:

$$Only\ Gujarati : 10\alpha = 50 \Rightarrow \alpha = \frac{50}{10} = 5$$

$$Only\ Marathi : 7\beta = 84 \Rightarrow \beta = \frac{84}{7} = 12$$

The equation of the hyperbola will be:

$$\beta^2 x^2 - \alpha^2 y^2 = \alpha^2 \beta^2 \Rightarrow \frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1 \Rightarrow \frac{x^2}{5^2} - \frac{y^2}{12^2} = 1$$

The values are $a = 5$ and $b = 12$.

The linear eccentricity will be:

$$c = \sqrt{a^2 + b^2} = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$$

The linear eccentricity of the hyperbola is 13.

Example 2: $\bigcup_{i=1}^{30} X_i = \bigcup_{i=1}^{n} Y_i = W$, where each X_i contains 5 elements and each Y_i contains 6 elements. If each element of the set W is an element of exactly 15 of sets $X_i's$ and exactly 3 of sets $Y_i's$, then evaluate $f = \frac{9}{\sqrt{n^2 - 16}}$.

Solution:

Since, $\bigcup_{i=1}^{30} X_i = \bigcup_{i=1}^{n} Y_i = W$

And, the number of elements in each set of X and Y is $n(X_i) = 5, n(Y_i) = 6$

So, the total number of elements will be:

$$\bigcup_{i=1}^{30} X_i = 30 \times 5 = 150, \bigcup_{i=1}^{n} Y_i = 6 \times n = 6n$$

Eliminate the repeating terms and solve to get the value of n as follows:

$$\frac{150}{15} = \frac{6n}{3} \Rightarrow n = \frac{3}{6} \times \frac{150}{15} \Rightarrow n = 5$$

The value will be:

$$f(5) = \frac{9}{\sqrt{5^2 - 16}} = \frac{9}{3} = 3$$

The value is $f(5) = 3$.

Example 3: Evaluate the number of functions g from the set $A = \{1,2,3\}$ into the set $B = \{0,1,2,3,4,5\}$ such that $g(l) \le g(m)$ for $l < m$ and l, m belong to A.

Solution:

As per the given condition, $g(0) \le g(1) \le g(2)$ and the function values will be from the set $\{0,1,2,3,4,5\}$. The values of set A will form the domain and the values in set B will form the range for the function. The number of possible cases will be:

Case 1: When the values of $g(0), g(1)$ *and* $g(2)$ are different, the possible ways are $^6C_3$.

Case 2: When two of the values in $g(0), g(1)$ *and* $g(2)$ are same, the possible ways are $2 \times {}^6C_2$

Case 3: When all the values $g(0), g(1)$ *and* $g(2)$ are same, the number of possible ways is $^6C_1$

The total number of possible ways are,

$$^{6}C_3 + 2\times {}^{6}C_2 + {}^{6}C_1 = {}^{6}C_3 + {}^{6}C_2 + {}^{6}C_2 + {}^{6}C_1 = {}^{7}C_3 + {}^{7}C_2 = {}^{8}C_3$$

The number of functions is $^{8}C_3$.

EXERCISE

1. Evaluate the sum of all the elements of the set $\{\alpha \in \{1,2,......,100\} : HCF(\alpha,45)=1\}$

2. In a study, the data of 500 persons was recorded. It was found that 135 persons experienced headache, 170 persons experienced cough, 240 persons experienced fever, 320 persons experienced headache or fever, 350 persons experienced cough or fever, 270 persons experienced headache or cough, and 25 persons experienced all three. If a person is chosen randomly from 500 persons, then what is the probability that the person has at most one symptom?

SOLUTIONS

1. The factors of the number are $45 = 3^2 \cdot 5$. For the H.C.F. to be 1 the value of α should be such that it is divisible neither by 3 nor 5. Let,

$$A : Divisible\ by\ 3 = \{3,6,9,.......,99\}$$
$$B : Divisible\ by\ 5 = \{5,10,15,....,100\}$$
$$A \cap B : Divisible\ by\ both\ 3\ and\ 5 = \{15,30,45,......,90\}$$

The number of elements in the two sets are $n(A)=33, n(B)=20$ and $n(A\cap B)=6$.

The sum of the elements in the set can be evaluated using the formula $S_n = \frac{n}{2}(First\ term + Last\ term)$

The sum will be:

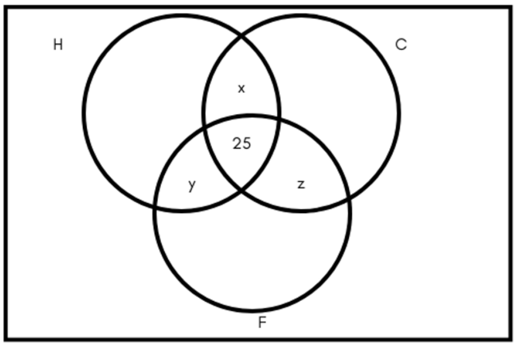
$$S_A = \frac{33}{2}(3+99) = 1683$$
$$S_B = \frac{20}{2}(5+100) = 1050$$
$$S_{A\cap B} = \frac{6}{2}(15+90) = 315$$
$$S_{A\cup B} = S_A + S_B - S_{A\cap B} = 1683 + 1050 - 315 = 2418$$

The total sum of all the elements in the set $\{1,2,3,......,100\}$ will be $S_T = \frac{100}{2}(1+100) = 5050$.

The sum of the elements divisible neither by 3 nor 5 is $S = S_T - S_{A\cup B} = 5050 - 2418 = 2632$

The required sum is 2632.

2. The Venn diagram can be sketched for reference as follows:



The number of people with at most one symptom is $500-(x+y+z+25)$.

Now,

$n(H\cup C): 135+170-(x+25) = 270$

$x = 135+170-25-270$

$x = 10$

Next, to find the value of y,

$n(H\cup F): 135+240-(y+25) = 320$

$y = 135+240-25-320$

$y = 30$

Similarly,

$n(F\cup C): 240+170-(z+25) = 350$

$z = 240+170-25-350$

$z = 35$

The people with at most one symptom are, $500-(10+30+35+25) = 400$.

The probability $P = \frac{400}{500} = 0.8$

The probability is 0.8.