Newton School of Technology

NSAT

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NSAT Exam Guidelines

Exam Protocol

- Arrive at least 15 minutes early (by 2:45 PM) for exam setup and proctoring checks.
- All three 3 sections are mandatory and include a sectional cutoff that determines passing eligibility
- The exam will be proctored. Unfair means will lead to permanent disqualification.
- For MCQ-type Questions, Positive marking will be (+4) for correct answers, while negative marking (-1) is applicable for incorrect ones.
- There is no negative marking in coding section.

Joining Details & Device Restrictions

- Use a laptop/PC with screen-sharing and microphone access
- Dual-camera setup required:
  - Primary camera (webcam) facing your face
  - Secondary camera (mobile/external) showing your workspace
- Mobile phones are only allowed as a secondary camera (not for taking the test)
- Calculators are prohibited; use pen and paper for rough work
- Access the test portal at https://my.newtonschool.co/nsat/timeline
- Accept the calendar invite sent to you on test day as a reminder
- Maintain a stable internet connection to prevent interruptions during NSAT
- Keep both cameras ON throughout the test
- Position cameras properly:
  - Primary camera: face clearly visible
  - Secondary camera: desk, hands, and screen visible
- Ensure the secondary camera is placed steadily (not handheld)

Preparation

- Visit NSAT homepage to check compatibility before the exam.
- Use Google Chrome for optimal performance.
- Bring a pen and paper for rough work during the exam.
- Attempt a mock test to understand the exam pattern, for practice, and to confirm your PC compatibility.

Environment Considerations

- Sit in a quiet environment with no background noise to minimize distractions and avoid disqualification.
- Ensure the lighting in the room is appropriate for clear visibility and comfortable reading and writing.

MATRICES AND DETERMINANTS

A matrix is a rectangular array of numbers (any system of numbers), arranged in rows and columns. If there are m rows, and n columns in a matrix, it is called an 'm by n' matrix, or a matrix of order m × n. The transpose of a matrix is obtained from A by changing its rows and columns, and columns into rows. It is denoted by A' or A^T.

Properties of matrix multiplication:

1. Non-Commutative: $AB \neq BA$ in general.
2. Associative: $(AB)C = A(BC)$
3. Distributive: $A(B+C) = AB + AC$
4. Scalar Multiplication: $k(AB) = (kA)B = A(kB)$
5. Transpose Property: $(AB)' = B'A'$

Symmetric Matrix: A matrix which is equal to its transpose, is a symmetric matrix. For a symmetric matrix $A = [a_{ij}], a_{ij} = a_{ji}$ for all i and j.

Skew Symmetric Matrix: A matrix A such that A = -A' is called skew symmetric. For a skew symmetric matrix $A = [a_{ij}], a_{ij} = -a_{ji}$ for all i and j.

Theorem 1: A square matrix can be uniquely expressed as a sum of a symmetric and a skew symmetric matrix.

Transformation of Matrices:

1. Row/Column Interchange: $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$. R_i, R_j stand for i^{th} and j^{th} rows and C_i, C_j stand for i^{th} and j^{th} columns.
2. Scalar Multiplication: $R_j \to kR_j$ or $C_j \to kC_j$; $k \neq 0$.
3. Row/Column Addition: $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$.

Inverse of a matrix: If A and B are two square matrices of the same order such that $AB = BA = I$, the unit matrix of the same order as A or B, then B is called the multiplicative inverse of or simply the inverse of A, written as A^{-1}. A is also called inverse of B, written as B^{-1}.

Theorem 2: Inverse of a matrix, if it exists, is unique.

Theorem 3: If A and B are invertible matrices of the same order, then

$$(AB)^{-1} = B^{-1}A^{-1}$$

Determinant of a square matrix:

$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei - afh + bfg - bdi + cdh - ceg$ is the determinant of the square matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$.

Properties of Determinants:

1. Swapping Rows/Columns: Changes sign of determinant
2. Identical Rows/Columns: $\det(A) = 0$
3. Row/Column Scaling: $\det(kA) = k^n \det(A)$ (for $n \times n$ matrix)
4. Row Addition: No change in determinant
5. Triangular Matrix: $\det(A)$ = Product of diagonal elements
6. Factor Property: If the determinant vanishes for $x = a$, then $(x - a)$ is a factor of $\det(A)$.

Singular matrix: A square matrix A is called singular if $\det A = 0$. If $\det A \neq 0$ the matrix A is called a non-singular matrix or a regular matrix.

Adjoint of a matrix: If $A = \left[a_{ij}\right]_{n\times n}$ is a square matrix, then the transpose of the matrix $\left[A_{ij}\right]_{n\times n}$ of which the elements are cofactors of the corresponding element in $|A|$, is called the adjoint of A. It is denoted by $adjA$.

Thus, $adjA = \left[A_{ij}\right]_{n\times n}$

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $adjA = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$ where A_{ij} is the cofactor of a_{ij} in $|A|$.

Theorem 4: If A is a square matrix, then $A(adjA) = |A|I = (adjA)A$.

Theorem 5: Let A be a square matrix. Then A^{-1} exists if A is non-singular and the inverse is given by

$$A^{-1} = \frac{1}{|A|} adjA$$

System of linear equations and solution:

Suppose we have the following system of equations:

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

We write $A=\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

In the matrix form becomes $AX=B$.

Theorem 6: The system of equations has a unique solution if and only if the matrix is non-singular, and in this case, the matrix solution of the equations is given by

$$X=\frac{1}{|A|}adjA\cdot B$$

Cayley-Hamilton Theorem: Every square matrix satisfies its characteristic equation.

$$A^n+c_1A^{n-1}+c_2A^{n-2}+....+c_nI=0$$

Eigenvalues and Eigenvectors:
Eigenvalue Equation: $Ax=\lambda x$
Characteristic Equation: $|A-\lambda I|=0$
Diagonalization: If A has n linearly independent eigenvectors, it can be written as $A=PDP^{-1}$ where P contains eigenvectors and D is a diagonal matrix of eigenvalues.

Rank of a Matrix:
Rank: Maximum number of linearly independent rows or columns.
Full Rank: If $rank(A)=\min(m,n)$, A has full rank.

Rank-Nullity Theorem: If A is a matrix of order $m\times n$, then

Rank of A + Nullity of A = Number of columns in $A=n$

LU Decomposition: A square matrix A can be decomposed as LU, where L is the lower triangular matrix and U is the upper triangular matrix.

Example 1: Let ω be a complex cube root of unity with $\omega\neq 1$ and $P=[p_{ij}]$ be a $n\times n$ matrix with $p_{ij}=\omega^{i+j}$. Then $P^2=0$, find the least prime number such that n should not be a multiple of that prime number.

Solution:

$$P=[p_{ij}]$$
$$p_{ij}=\omega^{i+j}$$

where ω is a complex cube root of unity such that $\omega\neq 1$, i.e., ω satisfies:

$$\omega^3=1, 1+\omega+\omega^2=0$$

$$(P^2)_{ij} = \sum_{k=1}^{n} P_{ik} P_{kj}$$

Substitute $P_{ik} = \omega^{i+k}$ and $P_{kj} = \omega^{k+j}$,

$$(P^2)_{ij} = \sum_{k=1}^{n} \omega^{i+k} \omega^{k+j}$$

$$(P^2)_{ij} = \sum_{k=1}^{n} \omega^{(i+j)+2k}$$

$$(P^2)_{ij} = \omega^{i+j} \sum_{k=1}^{n} \omega^{2k}$$

$$S = \sum_{k=1}^{n} \omega^{2k}$$

The sum $S = \sum_{k=1}^{n} \omega^{2k}$ is a geometric series with common ratio ω^2 and n terms. The sum of a geometric series is:

$$S = \frac{\omega^2(1-(\omega^2)^n)}{1-\omega^2}$$

For $P^2 = 0$, we require that this sum equals zero, i.e.,

$$\frac{\omega^2(1-(\omega^2)^n)}{1-\omega^2} = 0$$

which holds if and only if $(\omega^2)^n = 1$, or equivalently,

$$\omega^{2n} = 1$$

Since $\omega^3 = 1$, it follows that $\omega^{2n} = 1$ if and only if $2n$ is a multiple of 3, i.e.,

$$2n \equiv 0 \pmod 3$$

or equivalently,

$$n \equiv 0 \pmod 3$$

This means that for $P^2 = 0$, n must be a multiple of 3.

Example 2: How many 3×3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of $M^T M$ is 4?

Solution:

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$M^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Then,

$$M^TM=\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$M^TM=\begin{bmatrix} a^2+d^2+g^2 & ab+de+gh & ac+df+gi \\ ba+ed+hg & b^2+e^2+h^2 & bc+ef+hi \\ ca+fd+ig & cb+fe+ih & c^2+f^2+i^2 \end{bmatrix}$$

Let S be the sum of diagonal entries.

$$S=a^2+d^2+g^2+b^2+e^2+h^2+c^2+f^2+i^2$$

$$\Rightarrow S=a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2+i^2$$

Now, we will have to find the combinations of $S=4$ by putting the values of a,b,c,d,e,f,g,h and i as 0,1 and 2.

First, $4=1^2+1^2+1^2+1^2+0^2+0^2+0^2+0^2+0^2$

Second, $4=2^2+0^2+0^2+0^2+0^2+0^2+0^2+0^2+0^2$

In both the above cases, we have 9 entries.

In this first case, we have 4 elements of 1 and 5 elements of 0.

Therefore, the total number of combinations in the first case:

$$^9C_4=\frac{9!}{4!(9-4)!}=\frac{9\times8\times7\times6}{4\times3\times2\times1}=126$$

In the second case, we have one element of 2 and 8 elements of 0.

Therefore, the total number of combinations in the second case:

$$^9C_1=\frac{9!}{1!(9-1)!}=\frac{9}{1}=9$$

Hence, the total number of combinations is $126+9=135$.

Example 3: Let P be a matrix of order 3×3 such that all the entries in P are from the set {-1, 0, 1}. Then, find the minimum possible value of the determinant of P.

Solution:

$$P=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Therefore,

$$|P|=a\times\begin{vmatrix} e & f \\ h & i \end{vmatrix}-b\times\begin{vmatrix} d & f \\ g & i \end{vmatrix}+c\times\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

The maximum possibility of P can be 6 if we have the following:

$$\begin{vmatrix} e & f \\ h & i \end{vmatrix}=\pm2$$

$$\begin{vmatrix} d & f \\ g & i \end{vmatrix}=\pm2$$

$$\begin{vmatrix} d & e \\ g & h \end{vmatrix} = \pm 2$$

However, if $\begin{vmatrix} e & f \\ h & i \end{vmatrix} = \pm 2$

and $\begin{vmatrix} d & e \\ g & h \end{vmatrix} = \pm 2$, then

$\begin{vmatrix} d & f \\ g & i \end{vmatrix} = 0$

Therefore, $|P| \neq -6$.

Thus, the next possible value of $|P|$ is -4.

Example 4: Given $-x = cy + bz$, $-y = az + cx$, $-z = bx + ay$ where x, y, z, are not all zero. Find the value of $a^2 + b^2 + c^2 - 2abc$.

Solution:

Rewriting,

$$x + cy + bz = 0$$
$$cx + y + az = 0$$
$$bx + ay + z = 0$$

Using the condition for concurrency

$$\begin{vmatrix} 1 & c & b \\ c & 1 & a \\ b & a & 1 \end{vmatrix} = 0$$

$$1(1 - a^2) - c(c - ab) + b(ac - b) = 0$$
$$1 - a^2 - c^2 + abc + abc - b^2 = 0$$
$$a^2 + b^2 + c^2 - 2abc = 1$$

Example 5: Find the total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 14$.

Solution:

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 14$$

$$\Rightarrow x^3\begin{vmatrix}1 & 1 & 1+x^3\\ 2 & 4 & 1+8x^3\\ 3 & 9 & 1+27x^3\end{vmatrix}=14$$

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1$, we get

$$\Rightarrow x^3\begin{vmatrix}1 & 1 & 1+x^3\\ 0 & 2 & 6x^3-1\\ 0 & 6 & 24x^3-2\end{vmatrix}=14$$

$$\Rightarrow x^3(2(24x^3-2)-6(6x^3-1))=14$$

$$\Rightarrow x^3(12x^3+2)=14$$

$$\Rightarrow 6x^6+x^3=7$$

Put $x^3=t$,

$$\Rightarrow 6t^2+t-7=0$$

$$\Rightarrow t=\frac{-7}{6} \text{ or } t=1$$

$$\Rightarrow x^3=\frac{-7}{6} \text{ or } x^3=1$$

$$\Rightarrow x=\left(\frac{-7}{6}\right)^{\frac{1}{3}} \text{ or } x=1$$

So, x has 2 distinct values.

Example 6: Let k be a positive real number, and let $A=\begin{bmatrix}2k-1 & 2\sqrt{k} & 2\sqrt{k}\\ 2\sqrt{k} & 1 & -2k\\ -2\sqrt{k} & 2k & -1\end{bmatrix}$ and $B=\begin{bmatrix}0 & 2k-1 & \sqrt{k}\\ 1-2k & 0 & 2\sqrt{k}\\ -\sqrt{k} & -2\sqrt{k} & 0\end{bmatrix}$. If $\det(adjA)+\det(adjB)=729$, then find $[k]$.(Note: $adjM$ denotes the adjoint of a square matrix M and $[k]$ denotes the largest integer less than or equal to k)

Solution:

$|A|=(2k+1)^3, |B|=0$ ($\because B$ is a skew-symmetric matrix of order 3)

Let $\det(adjA)=|A|^{n-1}$

$$((2k+1)^3)^2=729$$

$$((2k+1)^3)^2=3^6$$

$$2k+1=3$$

$$k=1$$

$$[k]=1$$

EXERCISE

1. If $f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x\end{vmatrix}$, what is the least value of $f(x)$?

2. Let $P=\begin{bmatrix}1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1\end{bmatrix}$ and I be the identity matrix of order 3. If $Q=\left[q_{ij}\right]$ is a matrix such that $P^{100}-Q=I$, then calculate $\frac{q_{31}+q_{32}}{q_{21}}$.

3. Let A be a 3×3 real matrix such that $A^3-6A^2+11A-6I=0$ and $\det(A)=6$. Let $B=A^{-1}+2I$. Find $\det(B)$.

SOLUTIONS

1.

$$f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x\end{vmatrix}$$

$C_1 \to C_1+C_2$

$$f(x)=\begin{vmatrix}2 & \cos^2 x & 4\sin 2x \\ 2 & 1+\cos^2 x & 4\sin 2x \\ 1 & \cos^2 x & 1+4\sin 2x\end{vmatrix}$$

$R_2 \to R_2-R_1, R_3 \to R_3-R_1$

$$f(x)=\begin{vmatrix}2 & \cos^2 x & 4\sin 2x \\ 0 & 1 & 0 \\ -1 & 0 & 1\end{vmatrix}$$

$f(x)=2+4\sin 2x$

Thus, the least value of $f(x)$ is $f(x)=2-4=-2$. $(\because -1\le \sin 2x \le 1)$

2.

$$P=\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}$$

$$P^2=\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}$$

$$P^2=\begin{bmatrix}1&0&0\\3+3&1&0\\3^2+3^2+9&3+3&1\end{bmatrix}$$

$$P^3=\begin{bmatrix}1&0&0\\6&1&0\\27&6&1\end{bmatrix}\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}$$

$$P^3=\begin{bmatrix}1&0&0\\6+3&1&0\\27+18+9&6+3&1\end{bmatrix}$$

$$\therefore P^n=\begin{bmatrix}1&0&0\\3n&1&0\\\frac{n(n+1)}{2}9&3n&1\end{bmatrix}$$

Now, $Q=P^{100}-I$

$$Q=\begin{bmatrix}1&0&0\\300&1&0\\45450&300&1\end{bmatrix}-\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$Q=\begin{bmatrix}0&0&0\\300&0&0\\45450&300&0\end{bmatrix}$$

$$\therefore \frac{q_{31}+q_{32}}{q_{21}}=\frac{45450+300}{300}=152.5$$

The value obtained for $\frac{q_{31}+q_{32}}{q_{21}}$ is 152.5

3. $A^3 - 6A^2 + 11A - 6I = 0$

This is reminiscent of a minimal polynomial or characteristic polynomial.
Let's factorise:

$$x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$$

So, this suggests the eigenvalues of A are:

$$\lambda_1 = 1$$
$$\lambda_2 = 2$$
$$\lambda_3 = 3$$

Since the minimal polynomial has a distinct linear factor and is satisfied by A, we conclude: A is diagonalizable

Eigenvalues of $A : 1,2,3$

$B = A^{-1} + 2I$

Since eigenvalues of A are $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$, the eigenvalues of A^{-1} are $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}$

$$\mu_1 = 1$$
$$\mu_2 = \frac{1}{2}$$
$$\mu_3 = \frac{1}{3}$$

Then eigenvalues of $B = A^{-1} + 2I$ are:

$$\mu_1 + 2 = 3$$
$$\mu_2 + 2 = \frac{5}{2}$$
$$\mu_3 + 2 = \frac{7}{3}$$

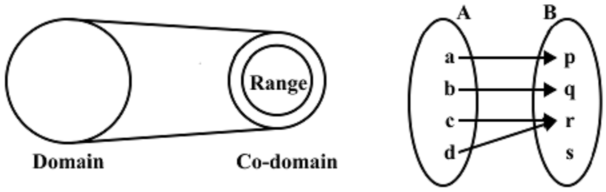
So,

$$\det(B) = \prod_{i=1}^{3} (\mu_i + 2) = 3 \times \frac{5}{2} \times \frac{7}{3}$$
$$\det(B) = \frac{35}{2}$$

The value of $\det(B)$ is $\frac{35}{2}$.

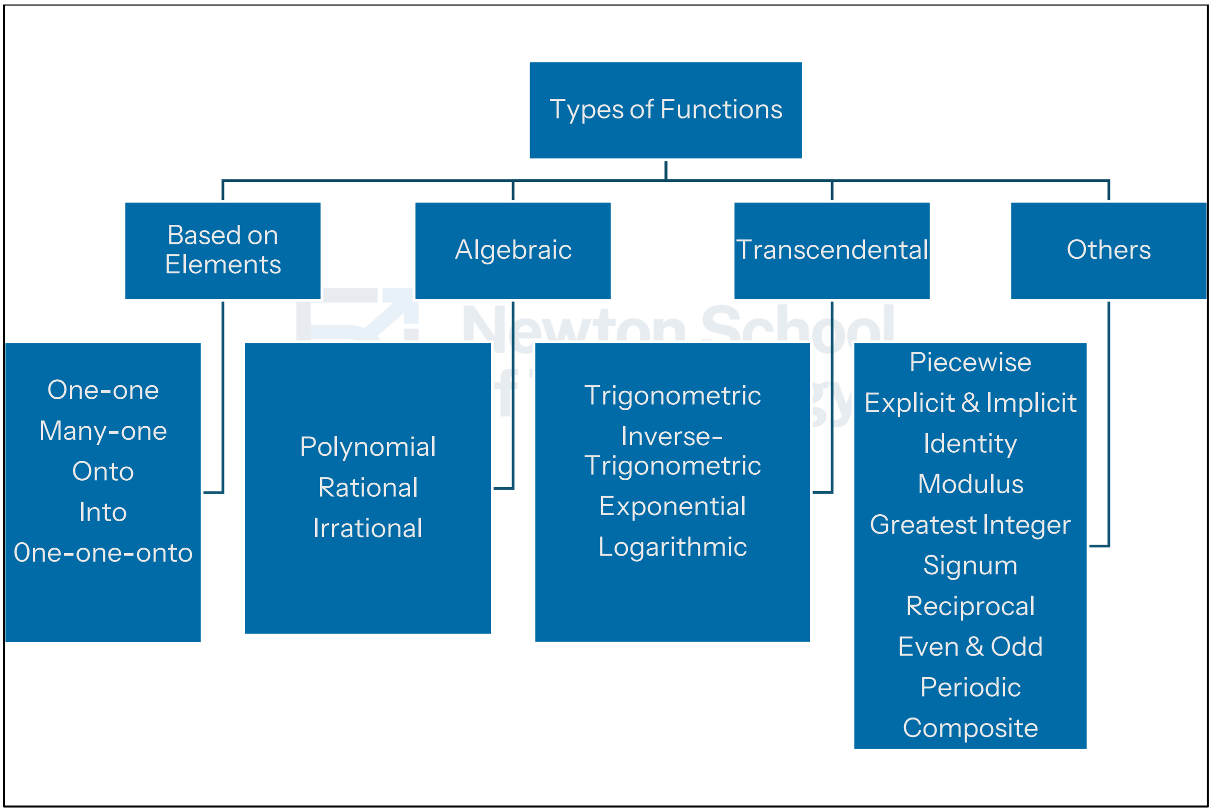
FUNCTIONS

Let A and B be two non-empty sets, then the rule f associated with each $x \in A$, with a unique number $y \in B$ is called a function from A to B. A function can be represented by mapping, by algebraic method, or in the form of an ordered pair. $f : A \to B$ is a function if each element in set A has its image in set B. For a function, it is impossible to have more than one image for a specific element in set A.



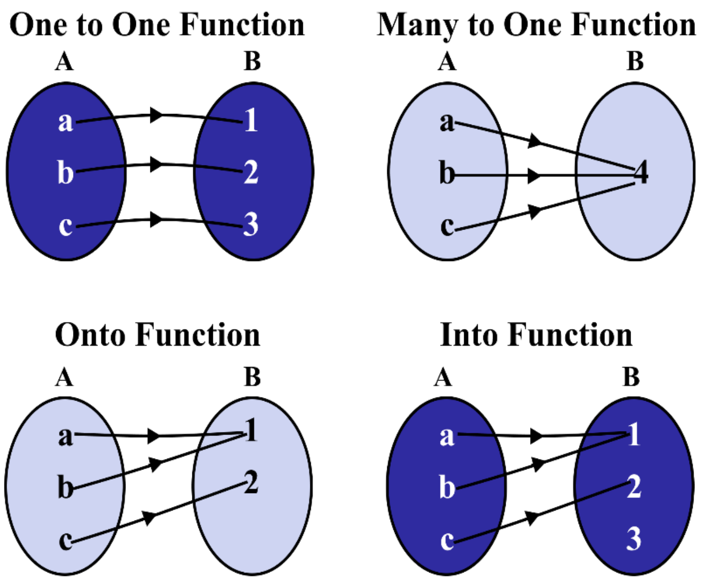
Domain, Co-domain and Range of a Function

Let the sets A and B have m and n elements respectively, then the total number of functions from set A to B is n^m.



One-one (injective) Function: If a function is given in the form of ordered pairs and if two ordered pairs do not have the same second element, then the function is one-one. If the graph of the function $y = f(x)$ is given, and each line parallel to x-axis cuts the given curve at most one point, then the function is one-one. If A and B are finite sets having m and n elements, respectively, then the number of one-one functions from A to B $= \begin{cases} {}^nP_m & \text{if } n \geq m \\ 0 & \text{if } n < m \end{cases}$

Many-one Function: If a function is given in the form of a set of ordered pairs and the second elements of at least two ordered pairs are the same, then the function is many-one. If the graph of $y=f(x)$ is given and the line parallel to the x-axis cuts the curve at more than one point, then the function is many-one.



Onto (Surjective) Function: It is a function where every element in the codomain (the set of potential outputs) is mapped to at least one element in the domain (the set of inputs). If A and B are two sets having m and n elements respectively, such that $1 \le n \le m$, then the number of onto functions from A to B is $\sum_{r=1}^{n}(-1)^{n-r}\,{}^{n}C_r r^m$.

Into Function: If range is a proper subset of the co-domain, then $y=f(x)$ is into.

One-one onto (bijective) Function: A bijective function is both injective and surjective. If A has n elements, then the number of bijections from A to B is the total number of arrangements of n items taken all at a time i.e. $n!$.

Algebraic Function: A function that consists of a finite number of terms involving powers and roots of the independent variable and the four fundamental operations. Ex. $x^5+6x^3+3, \dfrac{\sqrt{x}}{3x+2}$ etc.

Explicit Function: Expressed directly in terms of the independent variable. Ex. $y=f(x)$

Implicit Function: Cannot be expressed directly in terms of the independent variable. Ex. $x^2+y^2=c$

Greatest Integer Function: It is of the form $f(x)=[x]$ where $[x]$ is the integer equal to or less than x. Ex. $[4.2]=4, [-5.8]=-6$.

Modulus Function: It can be defined as $f(x)=|x|$ where the domain is the set of all real numbers and the range is the set of all non-negative real numbers.

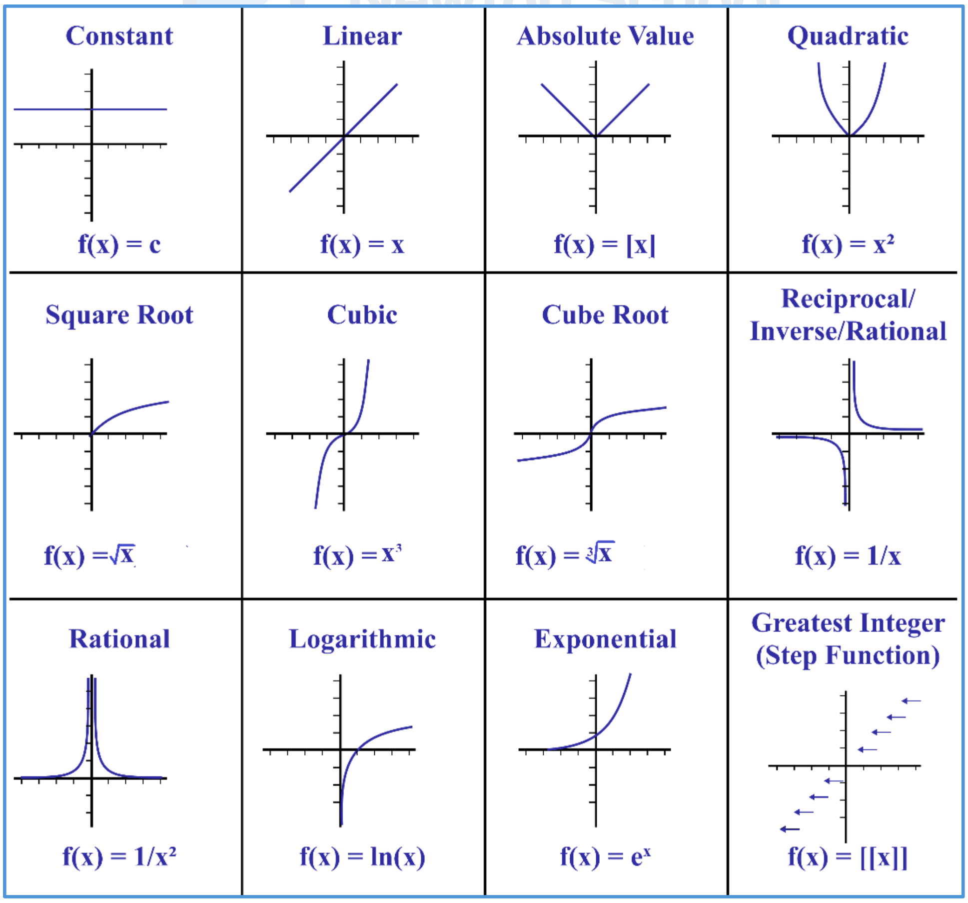
Signum Function: It can be defined as $f(x)=\begin{Bmatrix} \dfrac{|x|}{x} & \text{for } x \ne 0 \\ 0 & \text{for } x=0 \end{Bmatrix}$ where the domain is the set of all real numbers and the range is the set of -1,0 and 1.

Trancendental Function: Functions which are not algebraic. Ex: $\sin x, \cos^{-1} x, e^x, \log x$ etc.

Periodic Function: A function is said to be a periodic function if its each value is repeated after a definite interval. It is expressed as $f(x+T)=f(x)\forall x \in Domain$

Composite Function: It is the combination of two or more functions as a single function. Ex. $(f \circ g)(x)=f\{g(x)\}$

Graphical representation of different functions:



Properties regarding the Even $f(-x)=f(x)$ and Odd $f(-x)=-f(x)$ Functions

- Any polynomial function $f: R \to R$ is onto if the degree of f is odd, and into if the degree of f is even.
- An into function can be made onto by redefining the co-domain as the range of the original function.
- The graph of an odd function is always symmetric with respect to the origin.
- The product of two even functions is an even function.
- The sum and difference of two even functions is an even function.
- The sum and difference of two odd functions is an odd function.
- The product of two odd functions is an even function.
- The product of an even and an odd function is an odd function
- It is not essential that every function be even or odd. It is possible to have some functions that are neither even nor odd.
- The sum of even and odd functions is neither even nor an odd function.

- Zero function $f(x)=0$ is the only function that is even and odd both.

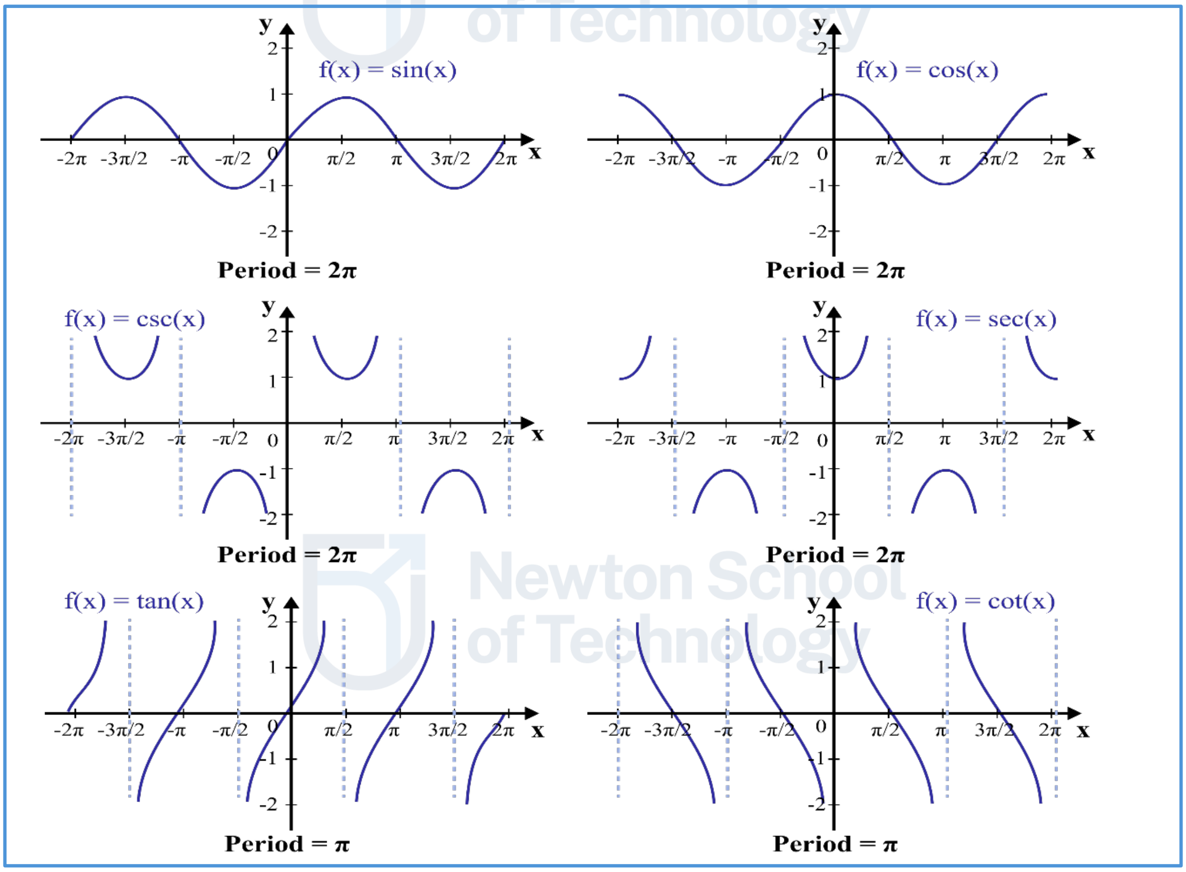
Trigonometric Function: The six trigonometric ratios (sin, cos, tan, cot, sec, csc) are the relationships between the angle and sides of a right triangle

Domain and Range of trigonometric and inverse trigonometric identitites

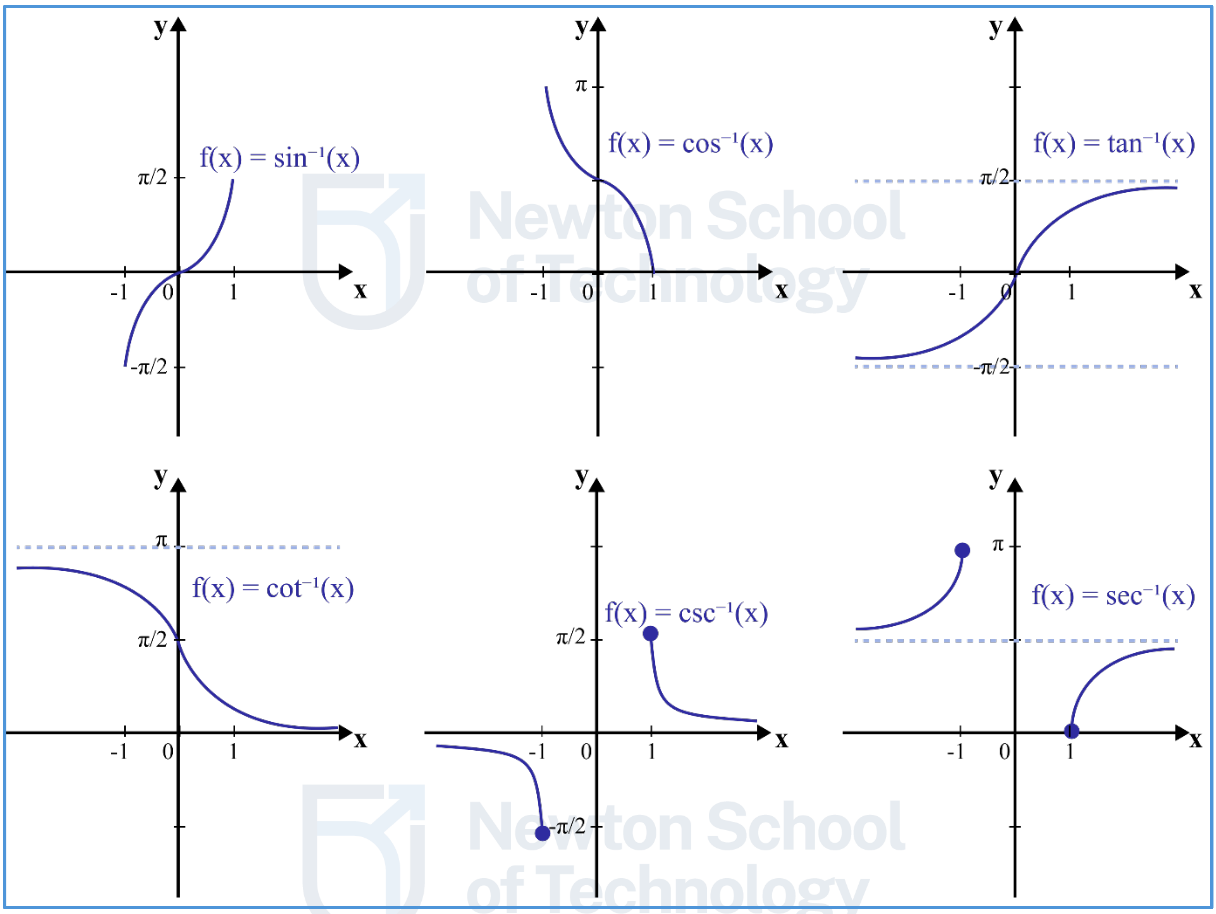
Trig Function	Domain	Range
$\sin x$	$\mathbb{R}$	$[-1,1]$
$\cos x$	$\mathbb{R}$	$[-1,1]$
$\tan x$	$\mathbb{R}-\left\{(2n+1)\frac{\pi}{2}, n\in \mathrm{I}\right\}$	$(-\infty,\infty)$
$\csc x$	$\mathbb{R}-\{n\pi, n\in \mathrm{I}\}$	$(-\infty,-1]\cup[1,\infty)$
$\sec x$	$\mathbb{R}-\left\{(2n+1)\frac{\pi}{2}, n\in \mathrm{I}\right\}$	$(-\infty,-1]\cup[1,\infty)$
$\cot x$	$\mathbb{R}-\{n\pi, n\in \mathrm{I}\}$	$(-\infty,\infty)$

Trig Function	Domain	Range
$\sin^{-1} x$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1,1]$	$[0,\pi]$
$\tan^{-1} x$	$(-\infty,\infty)$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
$\csc^{-1} x$	$(-\infty,-1]\cup[1,\infty)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$
$\sec^{-1} x$	$(-\infty,-1]\cup[1,\infty)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$
$\cot^{-1} x$	$(-\infty,\infty)$	$(0,\pi)$

Graph of Trigonometric Functions



Graph of Inverse trigonometric Function



Algebraic Operations on Function

Functions f and g with Domain f_D and g_D respectively
$(f+g)(x)=f(x)+g(x)\forall x\in D_f\cap D_g$
$(f-g)(x)=f(x)-g(x)\forall x\in D_f\cap D_g$
$(f\cdot g)(x)=f(x)\cdot g(x)\forall x\in D_f\cap D_g$
$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)};g(x)\neq 0,\forall x\in D_f\cap D_g$

List of Formulae Regarding Trigonometric Functions

Formula to transform the product into sum and difference	
$2\sin A\cos B=\sin(A+B)+\sin(A-B)$	$2\cos A\sin B=\sin(A+B)-\sin(A-B)$
$2\cos A\cos B=\cos(A+B)+\cos(A-B)$	$2\sin A\sin B=\cos(A-B)-\cos(A+B)$

Formulae to transform the sum or difference into a product	
$\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$	$\sin A-\sin B=2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$	$\cos A-\cos B=2\sin\frac{A+B}{2}\sin\frac{B-A}{2}$

Formulae for trigonometric ratios of the sum and difference of two angles	
$\sin(A+B)=\sin A\cos B+\cos A\sin B$	$\sin(A-B)=\sin A\cos B-\cos A\sin B$
$\cos(A+B)=\cos A\cos B-\sin A\sin B$	$\cos(A-B)=\cos A\cos B+\sin A\sin B$
$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$	$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$
$\cot(A+B)=\frac{\cot A\cot B-1}{\cot B+\cot A}$	$\cot(A-B)=\frac{\cot A\cot B+1}{\cot B-\cot A}$
$\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B=\cos^2 B-\cos^2 A$	
$\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B=\cos^2 B-\sin^2 A$	
$\tan A\pm\tan B=\frac{\sin(A\pm B)}{\cos A\cos B}$	$\cot A\pm\cot B=\frac{\sin(B\pm A)}{\sin A\sin B}$
$\tan A+\cot B=\frac{\cos(B-A)}{\cos A\sin B}$	$\tan A-\cot B=\frac{-\cos(B+A)}{\cos A\sin B}$
$1\pm\tan A\tan B=\frac{\cos(A\mp B)}{\cos A\cos B}$	$1+\cot A\cot B=\frac{\cos(A-B)}{\sin A\sin B}$
$1-\cot A\cot B=\frac{-\cos(A+B)}{\sin A\sin B}$	

Formula for the trigonometric ratios of multiples of an angle	
$\sin 2A=2\sin A\cos A=\frac{2\tan A}{1+\tan^2 A}$ ..	$\cos 2A=\cos^2 A-\sin^2 A=\frac{1-\tan^2 A}{1+\tan^2 A}$
$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$	$\tan 3A=\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}$
$\sin 3A=3\sin A-4\sin^3 A$	$\cos 3A=4\cos^3 A-3\cos A$
$\sin 4A=4\sin A\cos^3 A-4\cos A\sin^3 A$	$\cos 4A=8\cos^4 A-8\cos^2 A+1$
$\tan 4A=\frac{4\tan A-4\tan^3 A}{1-6\tan^2 A+\tan^4 A}$	$\sin 5A=16\sin^5 A-20\sin^3 A+5\sin A$
$\cos 5A=16\cos^5 A-20\cos^3 A+5\cos A$	

Formulae for the trigonometric ratio of submultiple angles
$\left\|\cos\frac{A}{2}+\sin\frac{A}{2}\right\|=\sqrt{1+\sin A}$
$\left\|\sin\frac{A}{2}-\cos\frac{A}{2}\right\|=\sqrt{1-\sin A a}$
$\tan\frac{A}{2}=\pm\sqrt{\frac{1-\cos A}{1+\cos A}}=\frac{1-\cos A}{\sin A}$, where $A\neq(2n+1)\pi$
$\cot\frac{A}{2}=\pm\sqrt{\frac{1+\cos A}{1-\cos A}}=\frac{1+\cos A}{\sin A}$, where $A\neq 2n\pi$

Formulae for conditional trigonometric identities	
$\sin^2 A+\sin^2 B+\sin^2 C=1-2\sin A\sin B\sin C$; If $A+B+C=90°$ then	
$\cos^2 A+\cos^2 B+\cos^2 C=2+2\sin A\sin B\sin C$; If $A+B+C=90°$ then	
$\sin 2A+\sin 2B+\sin 2C=4\cos A\cos B\cos C$; If $A+B+C=90°$ then	
If $A+B+C=180°$ then the following conditions are true	
$\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$	$\sin 2A+\sin 2B-\sin 2C=4\cos A\cos B\sin C$
$\cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cos B\cos C$	
$\cos 2A+\cos 2B-\cos 2C=1-4\sin A\sin B\cos C$	
$\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$	$\sin A+\sin B-\sin C=4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$
$\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$	$\cos A+\cos B-\cos C=-1+4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$
$\sin^2 A+\sin^2 B-\sin^2 C=2\sin A\sin B\cos C$	$\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$
$\cos^2 A+\cos^2 B-\cos^2 C=1-2\sin A\sin B\cos$	$\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$
$\cot B\cot C+\cot C\cot A+\cot A\cot B=1$	$\tan A+\tan B+\tan C=\tan A\tan B\tan C$
$\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}+\tan\frac{A}{2}\tan\frac{B}{2}=1$	$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

Trigonometric Equation	General Solution
$\sin\theta=\sin\alpha$	$\theta=n\pi+(-1)^n\alpha$ where $n\in I$
$\cos\theta=\cos\alpha$	$\theta=2n\pi\pm\alpha$ where $n\in I$
$\tan\theta=\tan\alpha$	$\theta=n\pi+\alpha$ where $n\in I$
$\sin\theta=0$	$\theta=n\pi$ where $n\in I$

$\cos\theta = 0$	$\theta = n\pi + \frac{\pi}{2}$ where $n \in I$
$\sin\theta = 1$	$\theta = 2n\pi + \frac{\pi}{2}$ where $n \in I$
$\sin\theta = -1$	$\theta = 2n\pi - \frac{\pi}{2}$ where $n \in I$
$\cos\theta = 1$	$\theta = 2n\pi$ where $n \in I$

Trigonometric Equation	General Solution
$\cos\theta = -1$	$\theta = (2n+1)\pi$ where $n \in I$
$\sin\theta = \pm 1$	$\theta = (2n+1)\frac{\pi}{2}$ where $n \in I$
$\cos\theta = \pm 1$	$\theta = n\pi$ where $n \in I$

Formulae for inverse trigonometric functions
$\cos^{-1}x+\cos^{-1}y=\begin{cases}\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right) \text{ if } -1\le x,y\le 1, x+y\ge 0\\ 2\pi-\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right) \text{ if } -1\le x,y\le 1, x+y\le 0\end{cases}$
$\cos^{-1}x-\cos^{-1}y=\begin{cases}\cos^{-1}\left(xy+\sqrt{1-x^2}\sqrt{1-y^2}\right) \text{ if } -1\le x,y\le 1, x\le y\\ -\cos^{-1}\left(xy+\sqrt{1-x^2}\sqrt{1-y^2}\right) \text{ if } -1\le y\le 0, 0< x\le 1, x\ge y\end{cases}$
$2\sin^{-1}x=\begin{cases}\sin^{-1}\left(2x\sqrt{1-x^2}\right) \text{ if } -\frac{1}{\sqrt{2}}\le x\le\frac{1}{\sqrt{2}}\\ \pi-\sin^{-1}\left(2x\sqrt{1-x^2}\right) \text{ if } \frac{1}{\sqrt{2}}\le x\le 1\\ -\pi-\sin^{-1}\left(2x\sqrt{1-x^2}\right) \text{ if } -1\le x\le-\frac{1}{\sqrt{2}}\end{cases}$
$3\sin^{-1}x=\begin{cases}\sin^{-1}\left(3x-4x^3\right) \text{ if } -\frac{1}{2}\le x\le\frac{1}{2}\\ \pi-\sin^{-1}\left(3x-4x^3\right) \text{ if } \frac{1}{2}\le x\le 1\\ -\pi-\sin^{-1}\left(3x-4x^3\right) \text{ if } -1\le x\le-\frac{1}{2}\end{cases}$
$2\cos^{-1}x=\begin{cases}\cos^{-1}\left(2x^2-1\right) \text{ if } 0\le x\le 1\\ 2\pi-\cos^{-1}\left(2x^2-1\right) \text{ if } -1\le x\le 0\end{cases}$
$3\cos^{-1}x=\begin{cases}\cos^{-1}\left(4x^3-3x\right) \text{ if } \frac{1}{2}\le x\le 1\\ 2\pi-\cos^{-1}\left(4x^3-3x\right) \text{ if } -\frac{1}{2}\le x\le\frac{1}{2}\\ 2\pi+\cos^{-1}\left(4x^3-3x\right) \text{ if } -1\le x\le-\frac{1}{2}\end{cases}$

Formulae for inverse trigonometric functions

$$\tan^{-1}x+\tan^{-1}y=\begin{cases}\tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & \text{if } xy<1\\ \pi+\tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & \text{if } x>0, y>0, xy>1\\ -\pi+\tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & \text{if } x<0, y<0, xy>1\end{cases}$$

$$\tan^{-1}x-\tan^{-1}y=\begin{cases}\tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & \text{if } xy>-1\\ \pi+\tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & \text{if } x>0, y<0, xy<-1\\ -\pi+\tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & \text{if } x<0, y>0, xy<-1\end{cases}$$

$$\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$$

$$2\tan^{-1}x=\begin{cases}\tan^{-1}\left(\dfrac{2x}{1-x^2}\right) & \text{if } -1<x<1\\ \pi+\tan^{-1}\left(\dfrac{2x}{1-x^2}\right) & \text{if } x>1\\ -\pi+\tan^{-1}\left(\dfrac{2x}{1-x^2}\right) & \text{if } x<-1\end{cases}$$

$$3\tan^{-1}x=\begin{cases}\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right) & \text{if } -\dfrac{1}{\sqrt{3}}<x<\dfrac{1}{\sqrt{3}}\\ \pi+\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right) & \text{if } x>\dfrac{1}{\sqrt{3}}\\ -\pi+\tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right) & \text{if } x<-\dfrac{1}{\sqrt{3}}\end{cases}$$

$$\sin^{-1}x+\sin^{-1}y=\begin{cases}\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) & \text{if } -1\le x,y\le 1, x^2+y^2\le 1\\ \pi-\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) & \text{if } 0<x,y\le 1, x^2+y^2>1\\ -\pi-\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) & \text{if } -1\le x,y<1, x^2+y^2>1\end{cases}$$

$$\sin^{-1}x-\sin^{-1}y=\begin{cases}\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right) & \text{if } -1\le x,y\le 1, x^2+y^2\le 1\\ \pi-\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right) & \text{if } 0<x\le 1, -1\le y\le 0, x^2+y^2>\\ -\pi-\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right) & \text{if } -1\le x<0, 0<y\le 1, x^2+y^2\end{cases}$$

Example 1: $f(x)=2\left[\cos^2 x+\cos^2\left(x+\frac{\pi}{6}\right)-\sqrt{3}\cos x\cos\left(x+\frac{\pi}{6}\right)\right]$ and $g\left(\frac{1}{2}\right)=\frac{5}{2}$, then find the value of $(gof)(x)$.

Solution

$$f(x)=2\left[\cos^2 x+\cos^2\left(x+\frac{\pi}{6}\right)-\sqrt{3}\cos x\cos\left(x+\frac{\pi}{6}\right)\right]$$

$$f(x)=2\cos^2 x+2\cos^2\left(x+\frac{\pi}{6}\right)-2\sqrt{3}\cos x\cos\left(x+\frac{\pi}{6}\right)$$

$$f(x)=1+\cos 2x+1+\cos\left(2x+\frac{\pi}{3}\right)-\sqrt{3}\cos\left(2x+\frac{\pi}{6}\right)-\sqrt{3}\cos\frac{\pi}{6}$$

$$f(x)=\left[2+\cos 2x+\cos\left(2x+\frac{\pi}{3}\right)-\sqrt{3}\cos\left(2x+\frac{\pi}{6}\right)-\frac{3}{2}\right]$$

$$f(x)=\left[\left(2-\frac{3}{2}\right)+2\cos\left(2x+\frac{\pi}{6}\right)\cos\frac{\pi}{6}-\sqrt{3}\cos\left(2x+\frac{\pi}{6}\right)\right]$$

$$f(x)=\left[\frac{1}{2}+\sqrt{3}\cos\left(2x+\frac{\pi}{6}\right)-\sqrt{3}\cos\left(2x+\frac{\pi}{6}\right)\right]$$

$$f(x)=\frac{1}{2}$$

The calculation for the composite function is as follows: $(gof)(x)=g\{f(x)\}=g\left(\frac{1}{2}\right)=\frac{5}{2}$.

Example 2: Let $f(a)=\left\{\frac{a^n}{(1+a^n)}\right\}^{1/n}$ for $n\geq 2$ and $g(a)=(\underbrace{fofo......of}_{f\text{ should be }n\text{ times}})(a)$. Then calculate $\int a^{n-2}g(a)da$.

Solution

Given $f(a)=\left\{\frac{a^n}{(1+a^n)}\right\}^{1/n}$, for $n\geq 2$. The composite function is calculated as follows:

$$(fof)(a)$$

$$=f(f(a))$$

$$=f\left[\left\{\frac{a^n}{(1+a^n)}\right\}^{1/n}\right]$$

$$=f\left[\frac{a}{(1+a^n)^{1/n}}\right]$$

$$=\frac{\frac{a}{\left(1+a^n\right)^{1/n}}}{\left[1+\left\{\frac{a}{\left(1+a^n\right)^{1/n}}\right\}^n\right]^{1/n}}$$

$$=\frac{a}{\left(1+2a^n\right)^{1/n}}$$

$$(fofof)(a)=f\left[\frac{a}{\left(1+2a^n\right)^{1/n}}\right]=\frac{\frac{a}{\left(1+2a^n\right)^{1/n}}}{\left[1+\frac{a^n}{\left(1+2a^n\right)}\right]^{1/n}}=\frac{a}{\left(1+3a^n\right)^{1/n}}$$

From the above two patterns we get $g(a)=\underbrace{(fofo......of)}_{f\text{ should be }n\text{ times}}(a)=\frac{a}{\left(1+na^n\right)^{1/n}}$

The calculation for the integral $\int a^{n-2}g(a)da=\int\frac{a^{n-1}}{\left(1+na^n\right)^{1/n}}da$. Let $u=1+na^n$ then $du=n^2a^{n-1}da$. Applying "u-substitution" the integral solution obtained

$$\int a^{n-2}g(a)da=\frac{1}{n^2}\int u^{-1/n}du=\frac{1}{n^2}\cdot\frac{u^{1-1/n}}{1-1/n}+c=\frac{\left(1+na^n\right)^{1-1/n}}{n(n-1)}+c.$$

Example 3: Find the sum $[1]+\left[1+\frac{1}{1000}\right]+\left[1+\frac{2}{1000}\right]+\left[1+\frac{3}{1000}\right]+........+\left[1+\frac{999}{1000}\right]$

,

Solution

As [*] denotes the greatest integer function, we have:

$$\left[1+\frac{999}{1000}\right]=[1.999]=1$$

$$\left[1+\frac{1}{1000}\right]=[1.001]=1$$

The solution is as follows:

$$=[1]+\left[1+\frac{1}{1000}\right]+\left[1+\frac{2}{1000}\right]+\left[1+\frac{3}{1000}\right]+........+\left[1+\frac{999}{1000}\right]$$
$$=1+1+.........+1(\text{thousand times})$$
$$=1000$$

Example 4: Find the domain of

$$f(x)=\log_2\left(-\log_{1/2}\left(1+\frac{1}{\sin\left(\frac{x^0}{10}\right)}\right)\right)+\sqrt{\log_2(\log_2 x)-\log_2(2-\log_2 x)-\log_2 5}$$

Solution

Applying the properties of rational and logarithmic functions

$$\log_2(\log_2 x)-\log_2(2-\log_2 x)-\log_2 5\geq 0$$

$$\log_2(\log_2 x)-\log_2 5(2-\log_2 x)\geq 0$$

$$\log_2\frac{\log_2 x}{(10-5\log_2 x)}\geq 0$$

$$\frac{\log_2 x}{(10-5\log_2 x)}\geq 2^0$$

$$\frac{\log_2 x}{(10-5\log_2 x)}\geq 1$$

Let, $\log_2 x=t$

So, $\frac{t}{(10-5t)}\geq 1\Rightarrow 5t\geq 10-t\Rightarrow 6t\geq 10\Rightarrow t\geq\frac{5}{3}$

So, $\log_2 x\geq 5\Rightarrow x\geq 2^5$

Again $\log_2 x>0\Rightarrow x>2^0\Rightarrow x>1$

Again $2-\log_2 x>0\Rightarrow\log_2 x<2\Rightarrow x<2^2$

From the above conditions, the domain is $2^2<x\leq 2^5$

Also,

$$-\log_{1/2}\left(1+\frac{1}{\sin\left(\frac{x^0}{10}\right)}\right)>0$$

$$\log_2\left(1+\frac{1}{\sin\left(\frac{x^0}{10}\right)}\right)>0$$

$$1+\frac{1}{\sin\left(\frac{\pi x}{180}\right)}>1$$

$$\sin\left(\frac{\pi x}{180}\right)>0$$

$$0<\frac{\pi x}{180}<\pi$$
$$0<x<180$$

Combining the intervals, the domain obtained for $f(x)$ is $2^2<x\le 2^5$.

Example 5:

Let $f(x)=\frac{25^x}{25^x+5}$, then find the value of the sum

$$f\left(\frac{1}{2025}\right)+f\left(\frac{2}{2025}\right)+f\left(\frac{3}{2025}\right)+\ldots\ldots+f\left(\frac{2024}{2025}\right)$$

Solution

$$f\left(\frac{1}{2025}\right)=\frac{25^{1/2025}}{25^{1/2025}+5}$$
$$f\left(\frac{2024}{2025}\right)=\frac{25^{2024/2025}}{25^{2024/2025}+5}=\frac{25\cdot 25^{-1/2025}}{25\cdot 25^{-1/2025}+5}=\frac{25}{25+5\cdot 25^{1/2025}}=\frac{5}{5+25^{1/2025}}$$

Now, adding the above expressions

$$f\left(\frac{1}{2025}\right)+f\left(\frac{2024}{2025}\right)=\frac{25^{1/2025}}{25^{1/2025}+5}+\frac{5}{5+25^{1/2025}}=1$$
$$f\left(\frac{2}{2025}\right)+f\left(\frac{2023}{2025}\right)=1$$
$$f\left(\frac{3}{2025}\right)+f\left(\frac{2022}{2025}\right)=1$$

.

.

$$f\left(\frac{1012}{2025}\right)+f\left(\frac{1013}{2025}\right)=1$$

The sum will be

$$f\left(\frac{1}{2025}\right)+f\left(\frac{2}{2025}\right)+f\left(\frac{3}{2025}\right)+\ldots\ldots+f\left(\frac{2024}{2025}\right)$$
$$=1+1+1\ldots\ldots+(1012\ \text{ times})$$
$$=1012$$

EXERCISE

1. Find the value of $\sum_{k=1}^{15} \frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{3}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{3}\right)}$

2. X and Y are two sets and $f: X \to Y$. If $\{f(c)=y; c \subset X, y \subset Y\}$ and $\{f^{-1}(d)=x; x \subset X, d \subset Y\}$, then find $\{f^{-1}(f(a))\}$.

3. An even polynomial function $f(x)$ satisfies a relation $f(2x)\left(1-f\left(\frac{1}{2x}\right)\right)+f(16x^2y)=f(-2)-f(4xy)\forall x,y \in R-\{0\}$ and $f(4)=-255$, $f(0)=1$, then find the value of $f(2)$

SOLUTIONS

1. Let $\theta_k = \left(\frac{\pi}{4}+\frac{k\pi}{3}\right); 0 \le k \le 15$. So, $\theta_k - \theta_{k-1} = \frac{\pi}{3}$

The calculation for the sum is as follows:

$$S = \sum_{k=1}^{15} \frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{3}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{3}\right)}$$

$$= \frac{2}{\sqrt{3}} \sum_{k=1}^{15} \frac{\frac{\sqrt{3}}{2}}{\sin\theta_{k-1}\sin\theta_k}$$

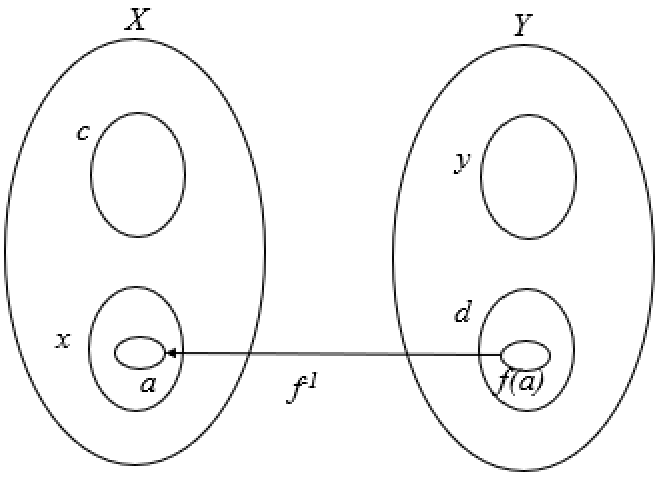
$$= \frac{2}{\sqrt{3}} \sum_{k=1}^{15} \frac{\sin\frac{\pi}{3}}{\sin\theta_{k-1}\sin\theta_k}$$

$$= \frac{2}{\sqrt{3}} \sum_{k=1}^{15} \frac{\sin(\theta_k - \theta_{k-1})}{\sin\theta_{k-1}\sin\theta_k}$$

Further, the calculation is as follows:

$$S=\frac{2}{\sqrt{3}}\sum_{k=1}^{15}\left(\cot\theta_{k-1}-\cot\theta_k\right)$$
$$=\frac{2}{\sqrt{3}}\left(\cot\theta_0-\cot\theta_{15}\right)$$
$$=\frac{2}{\sqrt{3}}\left[1-\cot\left(\frac{\pi}{4}+\frac{15\pi}{3}\right)\right]$$
$$=\frac{2}{\sqrt{3}}\left[1-\cot\left(5\pi+\frac{\pi}{4}\right)\right]$$
$$S=\frac{2}{\sqrt{3}}\left[1-\cot\frac{\pi}{4}\right]=2[1-1]=0$$

2. As per the conditions $f:X\to Y,\{f(c)=y;c\subset X,y\subset Y\}$ and $\{f^{-1}(d)=x;x\subset X,d\subset Y\}$ we can deduce the following:



As $f^{-1}(d)=x\Rightarrow f(x)=d;If\ a\subset x\Rightarrow f(a)\subset f(x)=d$

$\therefore f^{-1}(f(a))=a,a\subset x$

3. Let, $y=\frac{1}{8x^2}$. Substituting this value in the given relation

$$\Rightarrow f(2x)\left(1-f\left(\frac{1}{2x}\right)\right)+f\left(16x^2\cdot\frac{1}{8x^2}\right)=f(-2)-f\left(4x\cdot\frac{1}{8x^2}\right)$$
$$\Rightarrow f(2x)\left(1-f\left(\frac{1}{2x}\right)\right)+f(2)=f(-2)-f\left(\frac{1}{2x}\right)$$
$$\Rightarrow f(2x)\left(1-f\left(\frac{1}{2x}\right)\right)=-f\left(\frac{1}{2x}\right)\left[\because \text{"}f\text{"is an even function } f(-x)=f(x)\right]$$
$$\Rightarrow f(2x)+f\left(\frac{1}{2x}\right)=f(2x)f\left(\frac{1}{2x}\right)$$

If $f(x)$ is a polynomial satisfying the condition

$f(x)+f\left(\frac{1}{x}\right)=f(x)\cdot f\left(\frac{1}{x}\right)\forall x\in R-\{0\}$ then $f(x)=1\pm x^n$.

$f(2x)=1\pm(2x)^n\Rightarrow f(x)=1\pm x^n\Rightarrow f(4)=1\pm 4^n\Rightarrow -255=1\pm 4^n\Rightarrow -256=\pm 4^n$

$\Rightarrow -256=4^n$ (not possible); $-256=-4^n\Rightarrow n=4$

From the above pattern, we get $f(x)=1-x^n\Rightarrow f(2)=1-2^4=1-16=-15$

Newton School of Technology

NSAT

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NSAT Exam Guidelines

Exam Protocol

- Arrive at least 15 minutes early (by 2:45 PM) for exam setup and proctoring checks.
- All three 3 sections are mandatory and include a sectional cutoff that determines passing eligibility
- The exam will be proctored. Unfair means will lead to permanent disqualification.
- For MCQ-type Questions, Positive marking will be (+4) for correct answers, while negative marking (-1) is applicable for incorrect ones.
- There is no negative marking in coding section.

Joining Details & Device Restrictions

- Use a laptop/PC with screen-sharing and microphone access
- Dual-camera setup required:
  - Primary camera (webcam) facing your face
  - Secondary camera (mobile/external) showing your workspace
- Mobile phones are only allowed as a secondary camera (not for taking the test)
- Calculators are prohibited; use pen and paper for rough work
- Access the test portal at https://my.newtonschool.co/nsat/timeline
- Accept the calendar invite sent to you on test day as a reminder
- Maintain a stable internet connection to prevent interruptions during NSAT
- Keep both cameras ON throughout the test
- Position cameras properly:
  - Primary camera: face clearly visible
  - Secondary camera: desk, hands, and screen visible
- Ensure the secondary camera is placed steadily (not handheld)

Preparation

- Visit NSAT homepage to check compatibility before the exam.
- Use Google Chrome for optimal performance.
- Bring a pen and paper for rough work during the exam.
- Attempt a mock test to understand the exam pattern, for practice, and to confirm your PC compatibility.

Environment Considerations

- Sit in a quiet environment with no background noise to minimize distractions and avoid disqualification.
- Ensure the lighting in the room is appropriate for clear visibility and comfortable reading and writing.